how to use CIRCULAR LOG LOG SLIDE RULES

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PICKETT

THE WORLD'S MOST ACCURATE SLIDE RULES

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INDEX

WELCOME TO THE EXPANDING GROUP OF PICKETT SLIDE RULE USERS

Congratulations on receiving or purchasing your new Pickett circular slide rule. Pickett slide rules are known and used throughout the scientific, technical and educational communities of the world, including the programs of the National Aeronautical and Space Authority (NASA). Accurate to within 2 microns and using the exclusive yellow-green color of 5600 A to minimize eye fatigue, your circular slide rule has the quality of all Pickett all-metal slide rules.

If you are already familiar with the operation of an ordinary slide rule, you will find the operation of this circular slide rule easy to learn. Additionally, you will find it more convenient to use, since the scales are continuous and a setting is never "off scale". Finally, the circular nature of the scales enables you to conveniently carry the equivalent of a 13-inch slide rule on a disc which is little more than 2 inches in radius for the Model 109, or a 15 inch slide rule on a disk of 2-½ inches radius for the Model 111, while the expanded C and D scales of the Model 110 give the equivalent of a 50-inch slide rule on a disk with a diameter of 5 inches which easily slips into a coat pocket.

This manual covers the major operations which may be performed on the various scales of your Pickett circular slide rule. While this coverage is complete and thorough, it is necessarily terse. If you are learning to use the slide rule for the first time you may find the coverage of the manual rather brief. For additional explanations and practice problems, a slide rule beginner may want to refer to one of the large number of texts and paperbacked books that are available on the slide rule. These may be available in local book stores, or may be borrowed from public and school libraries. However, this manual does cover all the topics necessary for you to make complete use of your Pickett circular slide rule.

INTRODUCTION TO THE PICKETT CIRCULAR SLIDE RULE

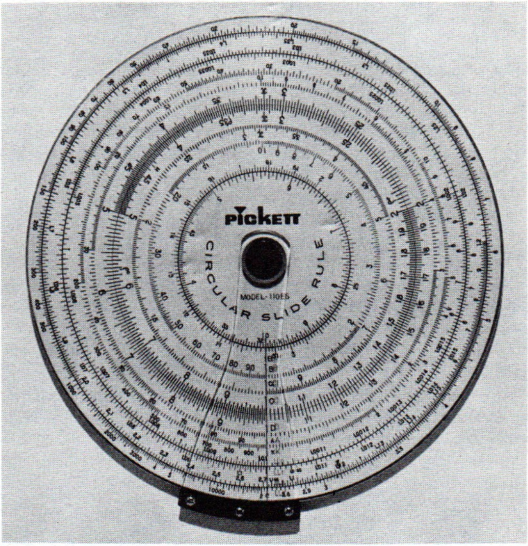
The plastic arm that is pivoted in the center of your circular slide rule is called the "cursor". On the cursor is a thin, red hairline which is used to align the various settings of the scales of the slide rule. The inner disk of your circular slide rule is best rotated by pressure from your thumbs as the slide rule is held in your hands as though it were a plate. The front face of the slide rule has the name PICKETT stamped on it. The reverse side of the slide rule is called the back face.

Each scale on the slide rule has an "index" or starting place. The scales are arranged so that all the indexes are aligned and marked with somewhat elongated lines. Each face and disk has an "Index Line." Along this index line, each of the scales is labeled with a letter of the alphabet. For convenience, the various scales are also printed on the plastic cursor alongside the hairline of the slide rule.

Your Pickett circular slide rule is designed so that all the index lines and both hairlines on the front and back faces of the slide rule may be simultaneously aligned. This is one test you can apply to insure that your hairlines and cursor are properly aligned.

READING THE SCALES

Hold your Pickett circular slide rule so that you are looking at the front face of the slide rule that has the word PICKETT stamped on the inner rotating disk. Rotate the inner disk until all the index marks and letters labeling all the scales are all aligned at the 6 o'clock (or bottom) position. The scale letters are all stamped in red for easy identification. The red hairline on the plastic cursor arm will help you align this "index line".

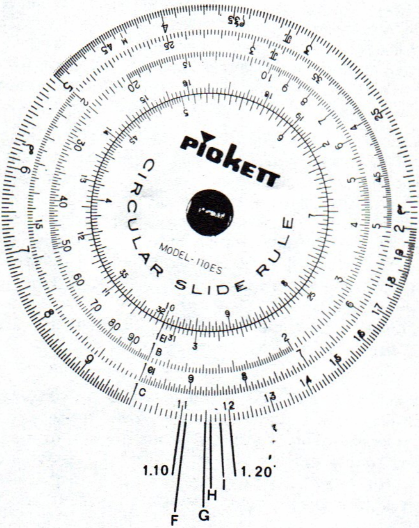


Note that all the scales are right-side-up and easy to read at the bottom of the slide rule. Thus you will want to rotate your circular slide rule so as to read the scales from the bottom of the rule.

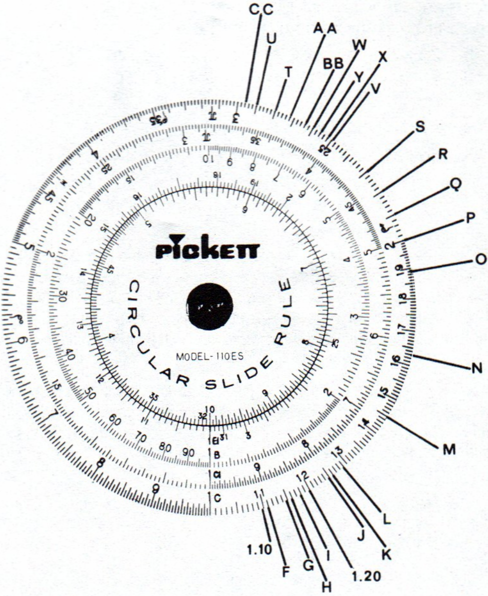
Learning to read the scales correctly is the first step in using your Pickett circular slide rule. All the scales of your circular slide rule are graduated into decimal parts. This means that all divisions are divided into equal parts of ten so that each mark stands for one tenth (.1), two tenths (.2), or five tenths (.5), of a whole unit. Since the C and D scales are the most used scales of the slide rule, we'll use them to demonstrate the reading of the scales.

Locate the C and D scales on your slide rule. If the index lines are properly aligned, you'll note that these two scales are identical. Keep them aligned this way as we cover their readings in this section. Just to the left of the letters C or D you'll see the large division mark which is labeled "1". This line is called the "Index" and marks the beginning and the end of each scale. On the D (and C) scale it stands for the numeral 1.00. Now swing your red hairline along to the right to the next large division which is marked as 11. Think of this as marking the value, 1.10. The next large mark of 12 would then be 1.20. We'll pause at these two marks for a moment.

In between 1.10 and 1.20 as shown, there are 10 spaces; each consecutive space is counted as one, two, three---and so on. The line labeled F in Diagram 2 is one division to the right of 1.10 and points to 1.11 and G points to 1.15. What are the readings for H and I in Diagram 2? Decide on these readings yourself before checking the answers given at the bottom of next page.



In Diagram 2, H = 1.16
I = 1.18



In a similar way in Diagram 3, J points to 1.25
K points to 1.26
L points to 1.29

What are the values of M, N. and O? Check your answers with those given at the bottom of this page.

Still in Diagram 3, the setting for P. is 2.00
" " " Q. is 2.10
" " " R. is 2.20

What are the readings for S, T. and U?

Now drop back to the values of V, W, and X in Diagram 3.

V is 2.50
W is 2.60
X is 2.52

This X reading is important. Note that between 2.50 (V) and 2.60 (W) there are five spaces; so each space is counted by two's, as two, four, six, eight. Thus X is 2.52 while Y is 2.56. The spaces beyond 2.50 are counted as two, four, six, giving us 2.56. What are the readings for AA, BB, and CC?

Looking at Diagram 4, now, note that the reading indicated by E is 3.36, F is 3.38. These two marks are separated by .02. To get a reading of 3.37, we would have to go just half way between E and F. The reading indicated by G is 3.47, just half

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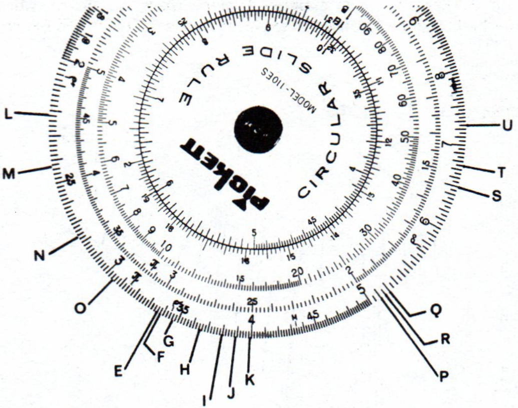
In Diagram 3, M = 1.45 S = 2.30
N = 1.63 T = 2.80
O = 1.89 U = 2.90

In Diagram 3, AA = 2.72
BB = 2.64
CC = 2.96

way between 3.46 and 3.48. The H reading is 3.65. What are the readings of I, J and K? (Correct readings are given at the bottom of the page.) Now compare your readings of the following with the answers given at the bottom of the page: L, M, N, and O.



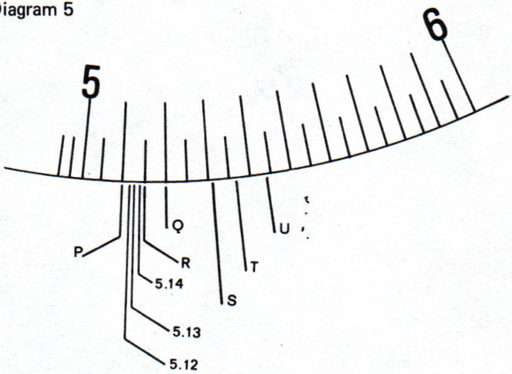
In Diagram 4,

I = 3.80	L = 2.22	S = 6.50
J = 3.89	M = 2.43	T = 6.75
K = 3.99	N = 2.77	U = 7.25
	O = 3.05	

In this same diagram,4, P = 5.10, Q = 5.20 and R, which is just half way between, indicates 5.15. What are your readings for S, T, and U?

Going back to P = 5.10 and R = 5.15, in the space between these two we can fit the numerals 5.11, 5.12, 5.13, and 5.14. One-fifth of the way from P to R would be 5.11, 2/5ths of the way would be 5.12, and so on. These distances between 5.10 and 5.15 have to be estimated, since there are no divisions marked on the scale. See the expanded scale in Diagram 5 below.

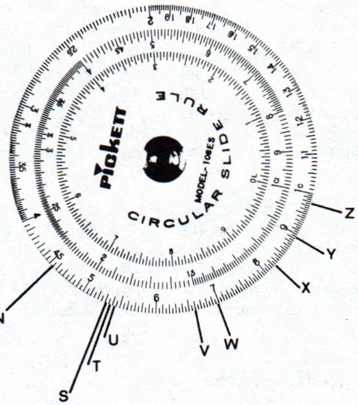
Diagram 5



Thus the reading 5.12 would be located at little less than half way between 5.10 and 5.15, these last two being the only marked divisions. The number 5.13 would be 3/5ths of the way, or a little more than half way from 5.10 to 5.15. The number 5.14 would be just to the left of 5.15. See if you agree with the following readings in Diagrams 5 and 6:

S = 5.31, T = 5.37, and U = 5.44

Diagram 6



Locate the following in Diagram 6, and check your readings with those given at the bottom of the page: N, V, W, X, Y. Z.

You may have noticed that all the numerals we have been working with thus far have been numerals lying between 1 and 10—for example 4.55 and 10.0, and so forth. Experience has shown that numerals in this format are easier for slide rule beginners to locate on the scales. Any numeral may be changed to fit this format in order to make it easy to locate on the slide rule scale. The numerals .455, 45,500, and 45.5 may all be changed to the format of a numeral between 1 and 10 by merely shifting their decimal points to give 4.55. And 4.55 we have already located as indicated by the letter N in Diagram 6. The numerals .455, 45,500 and 45.5 do NOT have the same value, but they are located at the same point of the slide rule scale, since they do have the same digits. Their different values due to their different decimal points may be taken care of as covered in the next section of this manual under PLACING THE DECIMAL POINT.

The reading of the scales—especially for a person learning to use the slide rule for the first time—requires a lot of practice until any numeral may be located accurately or read correctly without too much hesitation. That is why the accompanying diagrams have so many locations marked on them. By covering up the printed values you can test yourself constantly to insure

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(These letters in Diagrams 5 and 6)

N = 4.55	S = 5.31	W = 6.99
P = 5.10	T = 5.37	X = 8.23
Q = 5.20	U = 5.44	Y = 8.94
R = 5.15	V = 6.60	Z = 9.76

that the values you read from the scale are correct. This kind of practice must be done until you have confidence in your ability to properly read the scale.

All the scales on your slide rule are decimal scales, and may be read in the same way as you have learned to read the C and D scales. In approaching any of the scales to make a reading, always locate the divisions that have a value printed on the slide rule above and below the value you want to set. Then examine the divisions between these marked values to determine if they are in the tenths, two-tenths, or five-tenths, and then zero-in on the particular value you wish to set. The skill you gain in reading the slide rule scales will stand you in good stead in reading any other kind of a scale—for example, a multi-scaled electronic meter.

PLACING THE DECIMAL POINT

The reading of the scales of the slide rule has made you aware that the digits resulting from slide rule calculation give you no hint as to the placement of the decimal point in the answer. A variety of rules can be developed to aid in placing the decimal point, but only three systems will be mentioned here.

1. COMMON SENSE PLACEMENT OF THE DECIMAL POINT. If you are working a familiar problem, you know about the magnitude of the answer to expect, and you can place the decimal point accordingly.

2. THE ROUNDED-OFF CALCULATION. This system gives you the order of magnitude of the answer, and the more precise calculation on the slide rule gives you the digits to place in your answer. This is an easy system to use for relatively short problems. You merely round-off the numbers involved in your

problem until only one significant figure remains in each of the numbers.

For Example: $9.2 \times 47.6 \times 7.6 = ?$

This is an easy problem to set up on the slide rule, with the answer being read off the D scale as containing the digits 319. To get the decimal point, merely round off each of the three factors until they contain only one significant figure. This would give us 10 (rounded from 9.2) x 50 (rounded from 47.6) x 10 (rounded from 7.6), or, 10 x 50 x 10 = 5000. Our answer, then, should be approximately 5000. Our digits were 319. To bring the decimal point to give a value near 5000, we would give our more precise answer as being 3,190.

If the problem were $\frac{4960 \times 35.5}{51{,}000} = ?$

Set the numbers on the slide rule, work the problem, and get the digits 345 of the answer. To get the decimal point, round off the numbers to give you

$$\frac{5000 \times 40}{50{,}000} = \frac{40}{10} = 4$$

So, the more precise answer, read off your slide rule is 3.45 since this is close to 4, while 34.5 or .345 are not.

3. USE OF POWERS OF TEN. This is certainly the best manner of keeping track of the decimal point, no matter how involved or protracted a particular calculation may be. It is coming into widespread use, and is described in many textbooks of science and mathematics. In using powers of ten, all the numbers of a calculation are rewritten in a scientific notation format that has two parts; a decimal portion which lies between 1 and 10, and a power of ten. For example, the number 5,340

is rewritten as 5.34×10^3. Notice that the value 5.34 lies between 1 and 10 (or, more simply, has one digit to the left of the decimal point). Also, $10^3 = 10 \times 10 \times 10 = 1000$. So 5.34 x 1000 = 5,340 which was our original number. The exponent of the 10 (which is 3 in this case) tells us how far to move the decimal point—in this case, 3 places.

If our number is less than one, .0534, for instance, we change the .0534 to 5.34×10^{-2}. Here, the number is less than one, hence the exponent (-2 in this case) is negative. We moved the decimal point two places in going from .0534 to 5.34, and so the value of the exponent is 2. If this system of notation is new you'll want to study it more in detail using a suitable textbook.

After changing our numbers to powers of ten notation, we merely operate on the decimal number parts of the quantities using the slide rule. The powers of ten parts, using the algebraic rules of adding exponents in multiplication, subtracting exponents in division, multiplying exponents in raising to a power, and dividing exponents in extracting roots, enable us to easily and rapidly place the decimal point.

For example, in the problem $\frac{4960 \times 35.5}{51{,}000}$ we rewrite the terms in the powers of ten format (sometimes called scientific notation), and then multiply and divide by the digits shown, and use the rules above to operate on the exponents:

$$\frac{(4.96 \times 10^3) \times 3.55 \times 10^1)}{(5.1 \times 10^4)} = 3.45 \times 10^0 = 3.45$$

We added the exponents in the numerator, 3 + 1 = 4, and subtracted the exponent in the denominator, 4, from this total, giving us 0, and $10^0 = 1$.

It is beyond the scope of this booklet to provide a complete coverage of this system of notation and its use. But every serious student in science, mathematics, or technical work will find it in the literature and texts of his subject. Its use is strongly recommended. Lacking the background for the use of powers of ten or scientific notation, the method of section 2, above, may be used.

MULTIPLICATION

The C scale is used with the D scale for multiplication. To multiply any two numbers, such as 2 x 3, set the index line of the C scale over the first number, say 2, on the D scale; then move the cursor along the C scale until the hairline is over the second number, say 3, on the C scale. Directly below, under the hairline on the D scale you will read the answer 6.

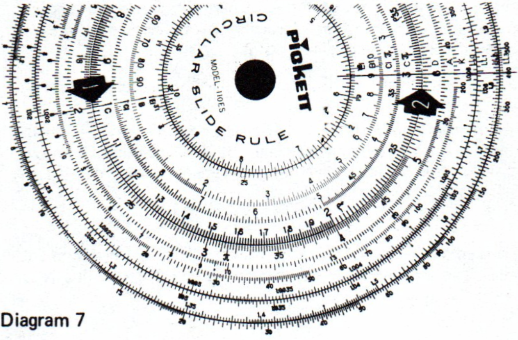


Diagram 7

RULE: Set the C index line over first factor on the D scale. Set the hairline over the second factor on the C scale. Read the product (answer) on the D scale under the hairline.

For example:

14.7 x 3.76 = ?	Set the hairline over 14.7 on the D scale. Rotate the inner disk until the C index line is under the hairline. This sets the C index line over 14.7 on the D scale. Now move the hairline to 3.76 on the C scale. Without disturbing this setting, or moving anything else, read the answer, 55.3 under the hairline on the D scale.
14.7 x 3.76 = 55.3	

Examples for you to try:

280 x 0.34 = 66.6
0.215 x 3.54 = 0.761

To multiply a series of numbers by a constant factor, as 3.14 x 35.6 = ?, 3.14 x 183 = ?, 3.14 x 0.23 = ? set the C index line over the constant factor, 3.14, on the D scale. Without disturbing this setting of the disk, move the hairline to each of the other numbers in turn along the C scale; at each hairline setting you may read the product on the D scale.

For continual multiplication, "r" x "s" x "t" x "u".....and so on: Set the C index line over the factor "r" on the D scale. Move the hairline to the factor "s" on the C scale. Without moving the hairline, now rotate the inner disk so that the C index is under the hairline. Now shift the hairline to the next factor, "t", on the C scale. Again rotate the disk until the C index is under the hairline, and move the hairline along the C scale to the next factor, "t", and so on. After setting the hairline on the last factor of the problem on the C scale, read the final product under the hairline on the D scale. (As you gain skill in using the CI scale, as covered below, this same problem can be simplified.)

DIVISION

The C and D scales are used for division. For simple division, say 6 divided by 3, move the hairline along the D scale until it is over the number being divided (dividend) say 6, on the D scale. Without disturbing the hairline, rotate the inner disk until the divisor, say 3, on the C scale is also under the hairline. Read the answer, 2, on the D scale, under the index of the C scale.

RULE: Set hairline at the dividend, quantity being divided by the divisor, on the D scale. Rotate the disk until the divisor (quantity being divided into the dividend on the C scale) is also under the hairline. Look for the quotient (answer) on the D scale under the index of the C scale.

For example:

56 ÷ 82 = 56/82 = ?	Set the hairline over 56 on the D scale. Rotate the inner disk until 82 on the C scale is also under the hairline. Read the answer under the C
56 ÷ 82 = 56/82 = 0.682	index on the D scale as being 0.682. You may find it easier to read this answer by moving the hairline over the C index and then reading the answer on the D scale under the hairline.

DIVISION BY A CONSTANT DIVISOR.

For example: 36/3.2 = ?
108/3.2 = ?
.24/3.2 = ?

Set the constant divisor, 3.2, on the C scale, over the index of the D scale. Move the hairline along the C scale to each of the dividends in turn, reading each of the quotients in turn on the D scale.

DIVISION BY A SERIES OF DIVISORS

For example: $\frac{45}{\text{"a"} \times \text{"b"} \times \text{"c"}}$

First divide 45 by "a" as explained above, then, instead of reading the quotient under the index of the C scale, move the hairline to the index of the C scale, and then without disturbing the hairline, rotate the disk to bring the second divisor, "b", on the C scale, under the hairline. Again, shift the hairline only to the index of the C scale. Now bring the next divisor "c", on the C scale, under the hairline by rotating the inner disk, and read the final answer (quotient) under the index of the C scale. Now bring the next divisor "c", on the C scale, under the hairline by rotating the inner disk, and read the final answer (quotient) under the index of the C scale. For additional divisors, continue this operation.

COMBINATION OF MULTIPLICATION AND DIVISION, USING C AND D SCALES

For example: $\frac{\text{"o"} \times \text{"p"} \times \text{"q"}}{\text{"r"} \times \text{"s"}}$

This problem can be solved most efficiently on the slide rule (that is, with the fewest possible settings) if you perform the initial division operation first, that is "o"/"r", and then multiply by "p" and then divide by "s", and finally multiply by "q".

For example:

$\frac{26 \times 7}{5} = ?$

We will divide 26 by 5 first and then multiply by 7. Set the hairline on 5 on the D scale, rotate the inner disk to bring 26 on the C scale under the hairline. (This gives you the result of dividing 26 by 5 if you read the answer on the D scale under the C index, but there is no need to make this extra reading.) To multiply by 7, now, and get your answer to the problem, merely leave the disk in its present setting, and move the hairline along the C scale to 7, and read your answer to the problem, 36.4, on the D scale under the hairline.

$\frac{26 \times 7}{5} = 36.4$

PROPORTIONS

Any direct or inverse proportion, once set up, can be easily solved using the C and D scales. In the proportion o/p = q/r, you merely set "o" on the C scale directly over "p" on the D scale. (Using the hairline will help you make a precise setting). Then, aligned with the value of "q" on the C scale will be the value of "r" on the D scale. Thus any ratio set up on the C and D scales will enable you to find the unknown quantity in any other equal ratio.

For example:

218/34 = 115/x

Set the hairline over 34 on the D scale. Rotate the inner disk until x = 17.94 — 218 is under the hairline. Move the hiarline to 115 on the C scale, and read the answer under the hairline on the D scale as 17.94.

A related problem would be

218/34 = y/25 — We have already aligned 218 of the C scale over 34 of the D scale in the problem above. To find "y", merely move the hairline to 25 on the D scale, and read "y" on the C scale as 160.5.

y = 160.5

You might notice that we are not solving these proportions in the usual sense of "product of the means equals the product of the extremes". Rather we are taking advantage of a mechanical setting of the C and D scales to get our answer directly, with a minimum of settings, regardless of whether the proportion is inverse or direct. This kind of a setting can often be used to an advantage in solving problems. For example, suppose a number of percentages need to be calculated from the same base, as shown below.

For example: An examination has 332 points. What are the percentage grades for scores of 300, 248, and 108?

Since 332 = 100%, this may be set up as:
332/100 = 300/r =
248/s = 108/t

Set the hairline over 100 on the D scale (which also happens to be the D index), and rotate the disk until 332 on the C scale is under the hairline. Now move the hairline to 300 on the C scale, and read r on the D scale as 90.4%. Shift the hairline to

332/100 = 300/90.4
r = 90.4%

332/11 = 248/74.7 s = 74.7%	248 on the C scale and read s as 74.7% under the hairline on the D scale. Finally, shift the hairline to
332/100 = 108/32.5 t = 32.5%	108 on the C scale to get t on the D scale as being 32.5%.

SQUARES AND SQUARE ROOTS

Squares and square roots may be easily determined on the slide rule with a single setting, using the D scale in conjunction with the A scale. In examining the A scale, as you follow it around to the right in the counter-clockwise direction you will notice that it has two parts; the first section is marked from 1 to 10, and then the second section runs from 10 to 100. This numbering of the two parts of the A scale helps you make a proper setting or reading on the scale.

The square of any number set on the D scale is found on the A scale.

For example:

$4^2 = ?$	Set the hairline on 4 on the D scale.
$4^2 = 16$	Under the hairline on the A scale read 16.
$25^2 = ?$	Set 25 on the D scale with the hairline,
$25^2 = 625$	and read 625 on the A scale.

To extract a square root, reverse this process by setting the number on the A scale, and reading the square root on the D scale. Since the A scale has two sections, as you've noticed, it is important to use the correct section of the A scale in setting your number.

For example:

$\sqrt{60} = 7.75$	Set the hairline over the division marked 60 on the A scale, and under the hairline on the D scale read the square root, 7.75.
$\sqrt{7} = 2.65$	Set the hairline over the division marked 7 on the A scale and under it, on the D scale, read the square root 2.65.

To properly set the slide rule to extract square roots, and to determine the placement of the decimal point in the root itself, it is only necessary to start at the decimal point of the original number and working both right and left (if necessary), point off pairs of digits. Let us assume our number is 6000. Since the decimal point is assumed to be after the right hand zero, after the digits have been paired off, it would look like this: 60 00. There will be one digit in the answer for each of the pairs of digits, so this pairing immediately tells us that this square root will have two places to the left of the decimal point. More than that, if the left hand "pair" has one digit the number is set in the 1-10 section of the A scale. If the left hand "pair" has 2 digits, it is set in the 10-100 section of the A scale. Our number 60 00 has 2 digits, "60", in its left hand "pair" and is set in the 10 to 100 section of the A scale under the hairline. To get 60 00, note the reading under the hairline on the D scale as 77.5.

Suppose the number had been 600.

$\sqrt{600} = ?$	1. Pair off numbers from the decimal point.
$\sqrt{600} =$	2. Left hand pair has one digit. Set 600 (at the 6) under hairline in 1-10 section of A scale.
$\sqrt{600} = \sqrt{600.} = 24.5$	3. Read the square root under hairline on D scale as 24.5.

Suppose the number had been .6.

$\sqrt{.6} = ?$	1. Pair off numbers from decimal point, adding zeros if necessary.
$\sqrt{.6} = \sqrt{.60}$	2. Left hand pair has 2 digits. Set it under hairline in 10-100 section of A scale at 60.
$\sqrt{.6} = \sqrt{.60} = .775$	3. Read square root on D scale as .775.

Additional examples:

$\sqrt{10.25} = 3.2$

$\sqrt{.00006} = \sqrt{.00775}$

$\sqrt{18{,}000} = \sqrt{1\ 80\ 00} = 134$

Selection of the proper section of the A scale is critical in extracting square roots. Until you gain experience, carefully double-check your A scale selection.

CUBES AND CUBE ROOTS

The K scale is used in conjunction with the D scale to cube numbers and to extract cube roots. Examine the K scale of your Pickett Circular Slide Rule, starting at its index line, which is labeled "1". It increases to the right (clockwise), with the major divisions being labeled 1, 2, 3...up to 10. Here, at 10, the scale repeats itself, with the major divisions being labeled 10, 20, 30,...up to 100. At 100, the scale again repeats itself, with the major divisions being labeled 100, 200, 300,...until the K index is reached again. Thus the K scale has three distinct sections (You'll remember that the A scale had two separate sections).

TO CUBE A NUMBER

If a number is set under the hairline on the D scale, its cube may be read directly on the K scale under the same hairline.

For example:

$2^3 = ?$	Set the hairline over 2 on the D scale, and read the cube, 8, on the
$2^3 = 8$	K scale.
$(3.3)^3 = ?$	Set the hairline over 3.3 on the D scale, and read the Cube, 35.9, on
$(3.3)^3 = 35.9$	the K scale.
$(8.8)^3 = ?$	Again, the setting of 8.8 on the D
$(8.8)^3 = 681$	scale gives the cube, 681, on the K scale. The digits in the cube, 186, are obtained from the K scale as before.
$(123)^3 = ?$	The decimal point in this example is determined by rough approxima-
$(123)^3 = 1{,}860{,}000$	tion, i.e., $100^3 = 1{,}000{,}000$.

TO EXTRACT CUBE ROOTS

The general method is similar to that used for square roots, except that for cube roots, "triplets" are grouped to the left and right of the decimal point instead of "pairs". If the left hand "triplet" has one digit, set it in the 1-10 section of the K scale. If it has 2 digits, set it in the 10-100 section of the K scale. Finally, if it has 3 digits as a full "triplet", set it in the 100-1000 section of the K scale.

For example:

$\sqrt[3]{3{,}300} = ?$

1. Separate the number into triplets from the decimal point.

$\sqrt[3]{3{,}300} = \sqrt[3]{3\ 300.}$

2. Left hand "triplet" has 1 digit. Set 3.3 in the 1-10 section of the K scale under the hairline.

$\sqrt[3]{3{,}300} = \sqrt[3]{3\ 300.} = 14.9$

3. Read the cube root under the hairline on the D scale as 14.9.

Another example:

$\sqrt[3]{.000512} = ?$

1. Separate into triplets, annexing a couple of zeros to fill out the last set.

$\sqrt[3]{.000\ 051\ 200} = ?$

2. Left group (052) has 2 digits, so set the number in the 10-100 section of the K scale under the hairline.

$\sqrt[3]{.0000512} =$

$\sqrt[3]{.000\ 051\ 200} = .0371$

3. Read the cube under the hairline on the D scale as .0371.

A final example:

$\sqrt[3]{.000512} =$

$\sqrt[3]{.000\ 512} = .08$

Here our triplets show us we are to use the 100 to 1000 (third) section of the K scale for our setting of 512, with cube root being read as .08 on the D scale.

USE OF THE CI SCALE

This scale, used in conjunction with the C and D scales, adds to the convenience of operation of the slide rule, allowing you to solve problems rapidly and efficiently with a minimum number of settings. The CI scale (which is pronounced as Cee-Eye) is found just above the C scale on the front side of the slide rule.

The CI scale is best investigated by aligning the index line of all your scales. Note that while readings on the C scale increase as you read to the right, or in a counter-clockwise direction, the CI scale increases as you read to the left in a clockwise direction. Thus the CI scale is "Inverted", or is the "Inverse", of the C scale. This arrangement immediately allows you to read on the CI scale the reciprocal of any number set on the C scale.

For example:

The reciprocal of
5 = 1/5 = ?

1/5 = .2

Swing the hairline along to C scale to 5. Immediately above read the reciprocal, .2, under the hairline on the CI scale.

The reciprocal of 8 = ?	Set the hairline at 8 on the C scale.
1/8 = .125	Read .125 on the CI scale.

This last setting will emphasize to you the need to remember that the CI scale is the INVERSE of the C scale, and is read from right to left, just the opposite of the C scale. This takes alertness and practice to insure that you read the proper values when using the CI scale.

The use of the CI scale will simplify many of the multiplication and division settings of your problems.

For example: Continued multiplication may be done with fewer settings using the CI scale in conjunction with the C and D scales.

Model Problem:

"r" x "s" x "t"

1. Set "r" on the D scale (as usual) with the hairline.
2. Under the hairline, set "s" using the CI scale.
3. Move the hairline to "t" on the C scale.
4. Read the answer on the D scale under the hairline.

Notice that in steps 1 and 2 you are really dividing "r" by 1/"s", which is the same as multiplying "r" by "s".

For example:

18 x 224 x 3.1 = ?

1. Set the hairline on 18 on the D scale.

2. Move the disk until 224 on the CI scale is under the hairline. (Remember to read the CI scale so the values increase to the left, or clockwise direction).

$18 \times 224 \times 3.1 = 12{,}500$

3. Move the hairline to 3.1 on the C scale, and find the answer, 12,500, under the hairline on the D scale.

Continued division may also be done with fewer settings.

Model Problem:

$$\frac{r}{s \times t} = ?$$

1. Using the hairline, set "s" on the C scale over "r" on the D scale (as in ordinary division).
2. Run the hairline along the CI scale to "t", and under it, read the answer on the D scale.

For example:

$$\frac{27}{15 \times 8.25} = ?$$

1. Using the hairline, set 15 on the C scale over 27 on the D scale.
2. Now shift the hairline to 8.25 on the CI scale, being careful to note that this setting is to the left (or clockwise) of the printed number 8, and read the answer .218 below it on the D scale.

$$\frac{27}{15 \times 8.25} = 0.218$$

THE EI SCALE

The EI (called Eee-Eye) is found on the front face of your Pickett circular slide rule as the innermost scale. It, similar to the CI scale, is an inverted or reciprocal scale which increases in a clockwise direction, but it is a doubled scale of 2 revolutions, and has graduations on both its inner and outer side. It starts at 1 at the index line and increases to the left, reading on the outer side, until it reaches 3.20 just before the index is reached after one complete circuit. As you continue reading this scale, clockwise, between 3.20 and 3.30 you pass the index line, and the readings shift to the inner side of the scale as you continue reading to the left in a clockwise direction of the second revolution of this scale.

Since this EI scale is a doubled CI scale, it contains the square root of any number set on the CI scale, and the CI readings represent the squares of the numbers set on the EI scale.

For example:

$\sqrt{17.0} = ?$ — Set the hairline on 17 on the CI scale, remembering that it is an inverted scale, and increases to the left, or in a clockwise direction.

$\sqrt{17.0} = 4.12$ — Read the square root under the hairline on the inner EI scale as 4.12.

Another example,

$\sqrt{4.40} = ?$ — Set hairline on 44 on CI scale. Read square root under hairline on outer EI scale as 2.098.

$\sqrt{4.40} = 2.098$

Just as in working with the A Scale, it is important to select the correct portion of the EI scale in reading the square root of the number set on the CI scale. If, after pointing off the pairs of digits as in the ordinary extraction of the square root there is a single digit, the square root is read on the outer portion of the EI scale; if there are two digits, the square root is read on the inner portion.

SPECIAL USES OF THE EI SCALE

Particular problems sometimes lend themselves to special settings on the slide rule which, in the interest of efficiency and to minimize the number of settings made, do not follow the regular sequence of multiplication, division, root extraction, and so forth. This is particularly true in the use of the EI scale of your Pickett circular slide rule. If you have developed some skill in using the B scale of an ordinary slide rule, you will find similar operations possible with the EI scale but with a greater precision, since the EI scale is a doubled scale, while the B scale of an ordinary slide rule is a halved scale. If, as a beginner in using the slide rule, you find the settings of this section confusing, please remember that these are special efficiency settings, and that each of these problems can be solved using the A, C, and D scales in the regular order of operations of the problem. It's just that the EI scale does the same job with a specialized procedure, with fewer settings, that's all.

Model Problem,

$\sqrt{rs}$ — Multiply "r" times "s", by setting "r" on the D scale with the hairline, and bring "s" on the CI scale under the hairline. Now shift the hairline to the D index line, and read the answer on the EI scale.

For example:

$\sqrt{60.5 \times 3.14} = ?$ — Set the hairline on 60.5 on the D scale. Rotate the disk to bring 3.14

on the CI scale under the hairline. Move the hairline to the D index line, and read the answer, 13.78 on the outside EI scale.

$60.5 \times 3.14 = 13.78$

As discussed earlier, care must be taken to determine which of the EI scales, inner or outer, is to be read. That the product within the radical in the previous example is about 180, or in the hundreds, indicates that the left hand pair would have only one digit, and so you would use the outer part of the EI scale, as covered previously. This rough estimate also indicates that there will be two digits to the left of the decimal point, since there are two paired grouping in 180, 1 80. Also, the square root of any number in the hundreds must lie between 10 and 100.

Model problem,

$\sqrt{r/s}$

Do this division, as usual, using the C and D scale, by setting "r" on the D scale under "s" on the C scale, using the hairline to assist in this alignment. Move the hairline to the D index line, and read the answer under the hairline up on the EI scale.

For example:

$\sqrt{13.45/38.3} = ?$

Set hairline over 13.45 on the D scale. Rotate the disk until 38.3 on the C scale comes under the hairline. Shift the hairline to the D index, and read the answer, 0.593 on the inner EI scale.

$\sqrt{13.45/38.3} = 0.593$

Model problem,

$rs^2 = ?$

Set "r" under the hairline on the D scale. Rotate the disk until "s" on the EI scale is under the hairline. Shift the hairline to the C index line and read the answer on the D scale under the hairline.

For example:

$2.43 \times (5.72)^2 = ?$

Set the hairline to 2.43 on the D scale. Rotate the disk until 5.72 on the EI scale is under the hairline Shift the hairline to the C index line and read the answer on the D scale as 79.5. A rough estimate sets the decimal point for you.

$(2.43)(5.72)^2 = 79.5$

Model problem,

r/s^2

Set the hairline on "r" on the D scale. Rotate the disk until the C index line is also under the hairline. Move the hairline to "s" on the EI scale, and read the answer under the hairline on the D scale.

For example:

$6.29/(5.11)^2 = ?$

Set 6.29 under the hairline on the D scale. Rotate the disk until the C index is under the hairline. Shift the hairline to 5.11 on the EI scale, and read the answer under the hairline on the D scale as 0.241.

$6.29/(5.11)^2 = .241$

Model problem,

r^2/s

(This is just the reciprocal of the preceeding model problem). Move the hairline to "s" on the D scale. Rotate the disk to bring the C index line under the hairline. Move the hairline to "r" on the EI scale and keep it there for the balance of the problem.

Without moving the hairline, align the C index line with the D index line, and then read the answer from the CI scale. (Note that since this is just the reciprocal of the preceeding type of problem, our final step was to take the reciprocal.)

For example:

$(3.56)^2/42.8 = ?$

Set hairline to 42.8 on the D scale and bring C index line under the hairline. Now move the hairline to 3.56 on the EI scale. Keeping the hairline fixed, rotate the disk until the C and D index lines are aligned. Now, read the answer on the CI scale as 0.296.

$(3.56)^2/42.8 = 0.296$

These models and examples illustrate some of the settings possible with the EI scale. Many others are possible — for example rs^2/t^2, $\sqrt{rs/t}$, and so forth. Some textbooks on the use of the slide rule list dozens of possible problems that may be solved in this efficient manner. Any procedure using the B scale of an ordinary slide rule may be used with greater precision on your Pickett circular slide rule with the EI scale.

TRIGONOMETRIC SCALES

The trigonometric scales are on the reverse side of the slide rule from the scales that have been covered thus far. Whereas the scales on the front side of the circular slide rule are best read at the bottom or six o'clock position, the trigonometric scales are best read at the top or 12 o'clock position. The trigonometric scales are marked as ST, S, or T, with S standing for angles whose sine values are to be read, and T standing for angles whose tangent values are to be read. Both sines and tangents can be read from the ST scale.

Smaller angles are on the inner scales, with larger angles on the outer scales. All angles are read as degrees and decimal parts of a degree. If an angle is given in degrees and minutes, it is necessary to change the minutes into decimal parts of a degree before setting the angle on the trig. scales. This is easily done by merely dividing the minutes by 60 and adding the decimal obtained to the whole number of degrees. For example, an angle of $60^\circ\ 36'$ would be set as $60^\circ + (36/60)^\circ$, or 60.6°.

SINE AND TANGENT FUNCTIONS

To find the value of the trigonometric function of an angle, set the hairline to the angle on an S scale for the sine function, and on a T scale for the tangent function. The value of the trigonometric function of the set angle is then read on the D scale. On Model 110 ES the D scale is not fixed, so the inner disk must be rotated until the trig. index line is aligned with the D scale index. Then the D scale is in position to read trig. functions.

On the innermost trig. scale, the tangent and sine functions are nearly essentially equal, so this ST scale is used for both functions. Move the hairline until it is set over 0.15° on the ST scale. The sine or tangent of 0.15° is read on the D scale as 0.00262. (Notice that on this innermost scale, the function read on the D scale is preceeded by two zeros and then the decimal point).

Now set an angle of 1.5° on the next ring. Note that the sine settings (S) are on the outer side of this ring, and the tangent settings (T) are on the inner side of this ring. Again, the functions are read on the D scale. Sin 1.5° = .0262. Also, Tan 1.5° = .0262. On this ring for angles from .58 to 5.7°, the D scale reading is preceeded by one zero and then the decimal point.

Now set Tan 15°. Note that the tangent scale for this angle is on the inner side of the third ring. Again the tangent of the angle is read on the D scale. Tan 15° = .268. On this third ring, where tangent angles run from 5.75° to 45°, the decimal point is placed to the left of the D scale reading.

Now set Sin 15°. This is also on the third ring, on the outer side. Read the function on the D scale, Sin 15° = .259. For sines set on the third ring, running from 5.75° to 90°, the decimal point is placed to the left of the D scale reading.

COSINE FUNCTIONS

To determine the cosine functions of angles, use is made of the relationship that the cosine of an angle is equal to the sine of the complement of the angle, or, cos X = sin $(90^\circ - X)$.

For example:

Cos 32.75° = ? Cos 32.75° = Sin 57.25° = ?	Find the complement of 32.75° by subtracting it from 90°, thus giving us an angle of 57.24°.
Cos 32.75° = Sin 57.25° = 0.841	Set the complement (57.24°) on the S scale, and read the function on the D scale as being .841.

RECIPROCAL TRIGONOMETRIC FUNCTIONS

From your experience thus far, you probably recognize the DI (pronounced Dee-Eye) scale as being an inverse D scale. Just as the CI scale and the C scale may be used to read reciprocal values, so may the DI and D scales. Since each of the trigonometric functions has a reciprocal, cosecant A = 1/sine A, secant A = 1/cosine A, and cotangent A = 1/tangent A, these reciprocal functions can be read directly on the DI scale.

On the Pickett circular slide rule Model 110 ES, the DI scale has been omitted to make room for other specialized scales, but reciprocal functions (cosecant, secant, and cotangent) may still be directly read from the trigonometric scales by aligning the two index lines of the back face of the slide rule. This automatically aligns the C and D index lines of the front side, allowing you to use the CI scale in place of the DI scale. Thus you can set an angle, say 32.75^{o} on the tangent scale on the back of the rule, and flip the rule over - making sure the index lines are aligned - and read the cotangent on the CI scale. If you are using Model 110 ES, use this system in place of the DI scale in the discussions that follow.

For example:

Cotangent 32^{o}45′ = ?	First change the minutes (45′) to a decimal fraction of a degree by calculating 45/60 using the C and D
Cotangent 32.75^{o} = ?	scales. Set the angle 32.75^{o} under the hairline on the T scale. Read the cot on the CI scale as being 1.555.
Cot 32.75^{o} = 1.555	The placement of the decimal point can be made by noting that the tan value would be in the 0.XXX range, hence the cot or reciproical would be in the X.XXX range.

Another example,

Csc 32.75^{o} = ?	Set the angle under the hairline on the S scale since CscA = 1/Sin A. Read the function on the DI scale
Csc 32.75^{o} = 1.85	as 1.85.

A final example:

Sec 50^{o} = 1/Cos 50^{o} = 1/Sin 40^{o} = ?	Since Sec A = 1/Cos A, and Cos A = Sin(90^{o}-A), set 40^{o} on the S scale, and read 1.555 on the DI scale from the bottom, or 6 o'clock position,
Sec 50^{o} = 1.555	of the slide rule.

SECANT, COSECANT, AND COTANGENT FUNCTIONS

Each of the trigonometric functions covered thus far has its reciprocal function, namely, cotangent A = 1/tangent A, cosecant A = 1/sine A, and secant A = 1/cosine A. Thus, to obtain these reciprocal functions, merely use the CI scale in conjunction with the D scale. To save resetting the function value on the front face of the slide rule after getting it on the D scale on the back face of the slide rule, merely align the C and D indexes on the front face. Then set the angle on the appropriate trigonometric scale. The direct function, sine, cosine, or tangent may then be read directly on the D scale on the back face of the slide rule, while the reciprocal function, cosecant, secant, or cotangent, may be read directly under the hairline on the front face of the slide rule on the CI scale. Be sure the C and D indexes on the front face are aligned in making this reading on the CI scale. This operation emphasizes that the index lines of the front and back faces of the slide rule are coincident, and also that the front and back face hairlines are fixed in alignment.

For example:

Cotangent 32°45' = ?	First change the minutes (45') to a decimal fraction of a degree by calculating 45/60 using the C and D scales.
Cotangent 32.75° = ?	Set the angle 32.75° under the hairline on the T scale. Align the C & D indexes on the front face. Read the cot on the CI scale on the front face
Cot 32.75 = 1.55	as being 1.555.

LOGARITHMS. THE L SCALE

To review logs briefly, a logarithm consists of two parts, a whole number, or integer, followed by a decimal portion. For example, 2.406. The integer is the "characteristic" (2 in our example), and is determined by the decimal placement in the original number. The decimal part of a logarithm is the mantissa, (.406 in our example), and is found by using the log scale (L scale) of the Pickett circular slide rule. A number greater than one has a positive characteristic which is one less than the number of digits to the left of the decimal point in the number. If the number is less than one, then the characteristic is negative and one unit more than the number of zeros following a decimal point before the first non-zero digit is reached.

To get the log to the base ten of any number, the log scale (marked "L") on the back face of the slide rule, is used in conjunction with the D scale. On the Model 110 ES the index lines of the L scale and the D scale are aligned to make log readings. Other models have the L and D scales already fixed in proper position. Set the number of the D scale with the hairline, and then read the mantissa of the log on the L scale. The mantissa always has a decimal point in front of the digits read on the L scale. The characteristic of the number is mentally determined by inspection and put to the left of the decimal point to obtain the complete log.

For example:

Log 255 = ?	Set 255 under the hairline on the D scale. (On Model 110 ES first align the L scale index with the D index line.) Read the mantissa off the L scale under the hairline as .406. Since the characteristic of
Log 255 = 2.406	255 is 2, the complete log is 2.406.

To find antilogs, reverse this process by ignoring the characteristic, and setting only the mantissa on the L scale under the hairline. On the D scale, whose index line is aligned with the L index, read the digits of the number you are seeking. Place the decimal point according to the characteristic of the log.

For example:

Antilog 3.885 = ?	Set .885 on the L scale, and read 767 on the D scale. Since the characteristic was 3, there will be 4 digits to the left of the decimal point, so the number would be 7,670.
Antilog 3.885 = 7,670 or Log 7,670 = 3.885	

GAUGE MARKS

Four different gauge marks have been printed over the C scale of your Pickett circular slide rule. These marks make the setting of particular values or constants easy in order to facilitate your calculations.

THE pi GAUGE MARK

This mark is found on both the C and D scales to facilitate the rapid setting of the value of pi as 3.1416 as it may occur in your calculations.

THE ρ^o, ρ', & ρ'' GAUGE MARKS

These are all found only on the C scale, and are used in converting radians to degrees, minutes, or seconds. In each case, the gauge mark is aligned with the D index, and a setting on the D scale in radians is converted in degrees, minutes, or seconds on the C scale, depending on the gauge mark used.

For example:

37.6^o = ? radians	Align the ρ^o gauge mark on the C scale with the D index. Move the

hairline to 37.6 on the C scale, and read the radians on the D scale as .656 radian. The decimal point is fixed by the fact that 57 degrees is about equal to 1 radian.

$37.6^{o} = 0.656$ radians

Another example:

1.3 radians = ? degrees

If the ρ^{o} gauge mark is aligned with the D index, move the hairline to 1.3 on the D scale and read under the hairline on the C scale the value of 74.5^{o}.

1.3 radian = 74.5^{o}

Using the ρ' gauge mark on the C scale aligned with the D index, radians on the D scale may be converted to minutes on the C scale, with the relationship of 1 minute being about .0003 radian being used to fix the decimal point.

The gauge mark ρ'' on the C scale aligned with the D index will convert radians on the D scale to seconds on the C scale, with one second being equal to about .000005 radian.

THE M GAUGE MARK

The M gauge mark is located at about .434 on the C scale. When it is set over the D index, logarithms to the base ten on the C scale may be converted to natural logarithms to the base e on the D scale. The decimal point may be fixed by the fact that a base 10 log is about .434 of a natural log.

For example:

Log_{10} 4.37 = .675
Log_e 4.37 = ?

Set gauge point M over the D index line. Move the hairline to 6.75 .675 on the C scale. Read the natural log on the D scale as 1.555.

Log_e 4.37 = 1.555

AREAS OF CIRCLES

Your Pickett circular slide rule has on the front face of its cursor a long red hairline which cuts across all the scales of the slide rule and a shorter red hairline off to the left that cuts only the A scale. This extra, short hairline enables you to calculate the area of any circle with a single setting.

RULE: Set the long hairline over the diameter of the circle on the D scale, and read the area of the circle on the A scale under the short, off-set hairline.

For example:

Diameter of circle = 3 in. Area of circle = ? Area of circle = 6.71 sq. in.	Set the long hairline over 3 on the D scale. Under the short, off-set hairline read the area on the A scale as 6.71.

The reverse of this procedure will give you the diameter of any circle whose area is known.

For example:

Area of circle = 21 sq. in. Diameter of circle = ? in. Diameter of circle = 5.17 in.	Set 21 on the A scale under the short off-set hairline. Read the diameter under the hairline on the D scale as being 5.17.

LOG LOG SCALES

Log log scales are found on Pickett circular slide rules Models 110 ES and 111 ES. These log log scales, which are lettered LL_0, LL_1, LL_2, LL_3, comprise what is really one continuous scale running in sequence, one section after another. On these four scales, there is only one location for a particular number, and the number is located complete with its decimal point. Thus the location of the number 10 is different from the location of the number 100. Also, the spacings between scale divisions are constantly changing, so that the readings must be carefully made. Fortunately, there are values printed at all the major divisions. In locating a number, always locate the printed number which is smaller, and the printed number which is larger than the one you are trying to set, and determine the values of the divisions marked between these two—some divisions have a value of only .00001, as on the LL_0 scale near the D index. In this same area, on the LL_3 scale, a division may have a value of 1.0000.

Readings on the log log scales are best made at the bottom or 6 o'clock position of the slide rule.

To give you some practice in making log log scale readings, move the hairline to the D index line on the front face of your slide rule. Now shift it a little to the right on the LL_3 scale to the division marked "3". This is for the number 3.00, and occurs no other place on the LL scales. Almost directly opposite it, on the far side of the slide rule, still on the LL_3 scale is the division marked 30. This emphasizes that you must find only one number in any one location on the LL scales, and you must include the decimal point.

Now, move the hairline back to 3.00 on the LL_3 scale. Without moving the hairline, read the value on the LL_2, LL_1, and LL_0 scales so that you can see the system used in each case. On the LL_2 you should read 1.1161. On LL_1 the reading is 1.01106. On LL_0 the reading is 1.0011, right on the division mark! Note that all the LL settings are greater than 1.

READING NATURAL LOGARITHMS

Using log log scales, natural logarithms can be directly obtained by merely setting the number on the LL or LL scale, and reading the natural logarithm on the D scale.

For example:

Log_e 15,000 = ?	Set 15,000 on the LL_3 scale under the hairline, and read the natural log
Log_e 15,000 = 9.62	directly on the D scale as 9.62.

The use of the LL scales determine natural logs and in this manner gives good precision if the numbers for which the natural logs are determined are on the LL_0, LL_1 and LL_2 or less than 4 on the LL_3 scale.

The placement of the decimal point may be made from your understanding of natural logarithms, or by applying the following rules:

LL_3 settings = X.XX on D scale
LL_2 settings = .XXX on D scale
LL_1 settings = .0XXX on D scale
LL_0 settings = .00XXX on D scale

EXPONENTIAL CALCULATIONS

The real power of the log log scales comes from their use in solving exponentials. If the base is more than 1 the LL scales are used. If the base is less than 1 the $\overline{LL}$ scales are used (These are only available on Model 111 ES).

RAISING TO A POWER (with exponents greater than 1):

$7.5^{1.26}$ = ?	Set the base on the log log scale, LL_3 in this case, under the hairline. Bring the C index up to the hairline. Move the hairline along the C scale in a counter-clockwise direction (to the right) to 1.26, and read the
$7.5^{1.26}$ = 12.6	answer 12.6 on the LL_3 scale.

You've probably recognized that what you are doing in this operation is multiplying the natural log of your base by its exponent.

$2.1^{1.6} = ?$

Set the hairline on 2.1 on the LL scales. It will be located on the LL_2 scale. Bring the C index up to the hairline. Move the hairline along the C scale counter-clockwise (to the right) to 1.6, noting that as you do so, the hairline moves past the D index line, indicating that your answer will be found on the next higher LL scale, or on LL_3. There, opposite 1.6 on the C scale read the answer 3.28 on the LL_3 scale.

$2.1^{1.6} = 3.28$

Note that whenever the hairline passes the D index or the LL index line (outlined in red), in moving along the C scale in a counter-clockwise direction to the value of the exponent, the answer will be found on the higher LL scale.

If the exponent is 10 or greater, the LL scale is automatically advanced one for the answer. If, in addition, the hairline passes the D index, in moving out along the C scale (counter-clockwise) the LL scale is advanced once more, in addition. If the exponent is up in the hundreds, the LL scale on which the answer is read is advanced two sections automatically.

For example:

$1.05^{120} = 350$

Base, 1.05, set on LL_1. Answer read on LL_3.

$1.05^{250} = ?$

The base, 1.05, is set on LL_1. However, the answer cannot be read directly, since the LL scale was advanced twice to take care of an exponent greater than 100, and then,

$1.05^{250} = (1.05)^{125} \times (1.05)^{125}\ (1.05)^{125}\ (1.05)^{125} = 445 \times 445 = 198{,}000$

in addition, the hairline crossed the index line in raising the base to the power, and we don't have an LL_4 scale. This problem may be broken down as shown and then each power may be determined separately, and then multiplied together for the final answer.

RAISING TO A POWER (with fractional exponents):

For example:

$81^{3/4} = ?$

First change the exponent into a decimal fraction by dividing the 3 by 4, using the C and D scales.

Now you have $81^{.75} = ?$

Set the hairline on 81 on the LL_3 scale. Bring the C index under the hairline by rotating the inner disk, and then move the hairline in a CLOCKWISE direction to the setting of .75 on the C scale. Notice that in this case the exponent is less than one, that is, it is a decimal fraction, and therefore the CLOCKWISE DIRECTION is used. If it passes the D index during this movement, the answer is read on the next lower LL scale. In this case, the D index line is not passed, and the answer, 27 is read on the same LL_3 scale.

$81^{.75} = 27$

For example:

$.81^{.075} = ?$

Here our procedure is the same as in the preceeding example, except we look for the answer on the LL scale with the next lower number, since the exponent has a zero following the decimal point. (If it had 2 zeros, we'd drop down two LL scales for our answer).

In the example we are now working, we set the hairline at .81 on the LL_3 scale, bring the C index under the hairline, and move the hairline in a clockwise direction (exponent is less than 1) to .075. The LL index is not passed, so our answer is found on the LL_2 scale as 1.390.

EXPONENTIAL e AS A BASE

Locate the LL index line, and note that on the LL_3 scale the number standing for exponential "e" coincides with the index line. Since the D index line and the LL index line also coincide, it is not necessary to align the C index when raising the base "e" to a power. Rather, the exponent may be set on the D scale with the hairline, and the answer then read directly on the LL scales under the hairline. On LL_3 if the exponent lies between 1 and 10, or LL_2 if it is between 1 and .1, LL_1 if it is between .1 and .01, and LL_0 if it is between .01 and .001, following the regular rules covered previously.

For example:

$e^{2.17} = ?$
$e^{2.17} = 8.76$

Set the hairline to 2.17 on the D scale, and read the answer as 8.76 on the LL_3 scale.

Another example:

$e^{.37} = ?$

Set .37 on the D scale under the

$e^{.37} = 1.448$	hairline, and read the answer, 1.448 on the LL_2 scale.

FINDING THE LOGARITHM OF A NUMBER TO ANY BASE

Our work in logarithms thus far has been to the base 10, or to the base e. Logarithms to any base may also be obtained.

For example:

$\log_2 8 = ?$	This expression may be read as: "What is the log of 8 to the base 2?"
$\log_2 8 = ?$, is the same as $2^x = 8$	This expression may be rewritten in exponential form, as shown. To solve this, merely set 2 under the hairline on the LL_2 scale. Rotate the disk until the C index is also under the hairline. Now move the hairline to 8 on the LL_3 scale. Under the hairline on the C scale, read
$2^3 = 8$, or $\log_2 8 = 3$	the exponent (or log) 3.

A similar problem may be one in which you know the number and its logarithm and you need to find the base of the logarithm.

For example:

$\log_x 16 = 4$, what is x? $\log_x 16 = 4$ is the same as: $x^4 = 16$	This problem may be rewritten in exponential form as shown on the left.
So, our problem is $x^4 = 16$	To solve this, set 16 on the LL_3 scale under the hairline. Rotate the disk until 4 on the C scale is also under the hairline. Move the hairline, now, to the C index, and read the
$2^4 = 16$, or $\log_2 16 = 4$	base on the LL_2 scale, as being 2.

THE $\overline{LL}$ SCALES
(Applies only to Model 111 ES)

The Pickett circular slide rule Model No. 111 ES has provisions for extending log log and exponential operations to numbers and bases less than one through the use of the $\overline{LL}$ scales on the back face of the slide rule. These $\overline{LL}$ scales are continuous, starting at a value of .000045 at the index line of $\overline{LL}_3$, and running through $\overline{LL}_2$, $\overline{LL}_1$, and $\overline{LL}_0$, which ends with a value of .99900.

To gain practice in reading this set of $\overline{LL}$ scales, hold your slide rule so that the red index line on the back face is in the 12 o'clock (or top vertical) position. Swing the hairline over to the 9 o'clock position, and there, on the $\overline{LL}_3$ scale, which is on the extreme outer edge of the slide rule, set the hairline precisely on the division marked 0.005. The division mark is right below the printed decimal point of the number. Keeping this same hairline setting, see what values you get for your readings under the hairline on the $\overline{LL}_2$, $\overline{LL}_1$, and $\overline{LL}_0$ scales.

The $\overline{LL}_2$ reading is 0.5585
The $\overline{LL}_1$ reading is 0.9484
The $\overline{LL}_0$ reading is 0.99472

READING NATURAL LOGARITHMS OF NUMBERS LESS THAN ONE

The natural logarithms of the numbers on the $\overline{LL}$ scales may be read directly on the D scale, just as the numbers of the LL scales were. However, since the $\overline{LL}$ scale numbers are all less than 1, their natural logarithms are all negative. The decimal point placement is given by the following:

$\overline{LL}_3$ settings = −X.XX on the D scale
$\overline{LL}_2$ settings = −0.XXX on the D scale

$\overline{LL}_1$ settings = −0.0XXX on the D scale
$\overline{LL}_0$ settings = −0.00XXX on the D scale

RECIPROCALS WITH LL AND $\overline{LL}$ SCALES

The numbers on the $\overline{LL}$ scales are the reciprocals of the numbers on the LL, and vice versa. To get the reciprocal of, say, 1.07, set the hairline on the front face of the slide rule to 107 on the LL_1 scale, flip the slide rule over (without moving the cursor or hairline), and read the reciprocal on the $\overline{LL}_1$ scale on the back face as being 0.9346.

Similarly, the reciprocal of 0.998 set under the hairline on the $\overline{LL}_0$ scale is read as 1.002 on the $\overline{LL}_0$ scale. Note that the number and its reciprocal appear on the same numbered log log scale, one on the LL_x scale and the other on the $\overline{LL}_x$ scale. You will recall that previously you used the C and CI scales to obtain reciprocals, but if the numbers are available on the three smaller (or three inner) log log scales, the results will be much more precise.

OPERATIONS WITH THE $\overline{LL}$ SCALES

All the operations of the LL scales previously covered, apply to the $\overline{LL}$ scales. Since the $\overline{LL}$ scales are on the back face of the slide rule, it is necessary to flip the slide rule over to use the C scale in exponential operations. However, the hairlines and the indexes of the two sides of the slide rule are aligned to facilitate these operations. The same rules as used in the LL scales apply to the $\overline{LL}$ scales. Some examples will show this.

For example:

$0.75^{2.7} = ?$ Find .75 on the $\overline{LL}_2$ scale and set it under the hairline. Flip the slide rule over to the front face without disturbing the setting of the hairline, and rotate the inner disk until the

C index is under the hairline. Now move the hairline along the C scale in a counter-clockwise direction (since the exponent is greater than 1) to the setting 2.7. Note that the log-log-index line is not passed in this case, so that our answer will be found on the same LL scale as our original setting, namely $\overline{LL}_2$ on the back face of the slide rule. So, flip the slide rule over to the back face without changing the last setting of the hairline and read the answer, 0.46 under the hairline on the $\overline{LL}_2$ scale.

$.75^{2.7} = 0.46$

For example:

$.75^4 = ?$

Set the hairline on $\overline{LL}_2$. Flip the slide rule over to the front face and bring the C index under the hairline. Move the hairline along the C scale in a counter-clockwise direction to 4. Note that you pass the LL index line, so you will read your answer on the next higher numbered $\overline{LL}$ scale, or on the $\overline{LL}_3$ scale, as being 0.3158.

$.75^4 = .03158$

For example:

$.75^{2.4/8} = ?$
$.75^{.3} = ?$

Divide the exponent to get a decimal exponent. Set the hairline to .75 on $\overline{LL}_2$, flip the slide rule and align the C index with the hairline. Now move the hairline in a clockwise direction along the C scale to

.3. (Clockwise, because the exponent is less than 1). Note that the LL index is passed in reaching .3, so the answer will be read on $\overline{LL}_1$, the next lower numbered $\overline{LL}$ scale, as 0.9174.

$75^{.3} = 0.9174$

These few examples show that the procedures used on the LL scales, as previously covered, apply to the $\overline{LL}$ scales as well.

EXPONENTIALS WITH NEGATIVE POWERS

The reciporical repicrocal reciprocal relationship between the LL and $\overline{LL}$ scales allows the easy treatment of negative powers.

For example:

$1.25^{-3.6} = ?$

Set the hairline on 1.25 on the LL_2 scale. Bring the C index under the hairline. Move the hairline counterclockwise (3.6 is more than 1, ignoring the negative sign for the moment) to 3.6. The LL index is not passed, so the answer will be found on the original numbered LL scale. Flip the slide rule to the back face, and read the answer, 0.448, on the $\overline{LL}_2$ scale. If the LL index line had been passed in getting to 3.6, the $\overline{LL}_3$ scale would have been read for the answer. What we are really doing here is getting the solution to $1.25^{3.6}$, and then taking the reciprocal.

$1.25^{-3.6} = 0.448$

As a final example:

$e^{-4.2} = ?$

Set the hairline on 4.2 on the D scale on the front face of the slide

$e^{-4.2} = 0.015$

rule. Flip the slide rule over and read the answer on the $\overline{LL_3}$ scale on the back face of the slide rule, under the hairline, as 0.015.

SPECIAL SCALES FOUND ONLY ON THE MODEL 110 ES SLIDE RULE

All the scales and operations described in the balance of this manual apply only to the Pickett circular slide rule Model No. 110 ES.

THE V_1 AND V_2 SCALES

The V_1 and V_2 scales are on the rear face of the Model 110 ES slide rule at the interface of the inner disk and the outer part of the rule. These are best read at the six o'clock position. The V_1 and V_2 scales are not logarithmic and may only be used together, and only for the solution of right triangles, or problems of a similar form, that is, $c = \sqrt{a^2 + b^2}$ or $b = \sqrt{c^2 - a^2}$.

For example:

$c = \sqrt{30^2 + 40^2} = ?$

Set the index (marked with a red triangle) of the V_1 scale over the first number, 30, on the V_2 scale. Move the hairline along the V_1 scale in a counter-clockwise direction to the second number, 40. Read the answer, under the hairline, on the V_2 scale, 50.

$c = \sqrt{30^2 + 40^2} = 50$

Now try

$c = \sqrt{50^2 + 90^2} = ?$

Set the V_1 index over 50 on the V_2 scale. Now move the hairline along the V_1 scale in a counter-clockwise direction to 90 on the V_1 scale. As you do this, notice that you pass the V_2 index, and are starting to repeat the V_2 scale. Since this scale is NOT logarithmic, you cannot go around and pass the index line. Rather, you have to re-state the problem by altering the decimal points until the answer is "on scale". Do this by re-stating the problem as shown on the left.

$c = \sqrt{5^2 + 9^2} = ?$

Set the V_1 index over 5 on the V_2 scale (this will be the first graduation marked on the V_2 scale), move the hairline along the V_1 scale in a counter-clockwise direction to 9, and read the V_2 scale as 10.3. Move the decimal point back again, and you have the answer 103.

$c = \sqrt{5^2 + 9^2} = 10.3$

So, $c = \sqrt{50^2 + 90^2} = 103$

Solving for the leg of a right triangle is also possible on the V_1 and V_2 scales.

For example:

$\sqrt{5.7^2 - 4.5^2} = ?$

Set the hairline on 57 of the V_2 scale. Rotate the inner disk until 45 on the V_1 scale is under the same hairline. The reading on the V_2 scale, at the V_1 index is 35, giving an answer of 3.5.

$\sqrt{5.7^2 - 4.5^2} = 3.5$

You may want to practice with the following additional problems:

$\sqrt{2.74^2 + 4.63^2} = 5.38 \qquad \sqrt{71.2^2 - 32.4^2} = 63.4$

THE B SCALE
(Applies only to Model 110 ES)

The B scale is found on the inner disk of the front side of the slide rule. It has the same graduations as the A scale, and may be used for the multiplication of squares and square roots in problems covered under the discussion of the use of the EI scale. As a matter of fact, the use of the EI scale results in more precise results, but sometimes the B scale may prove to be more convenient.

Model Problem,

$r/\sqrt{s} = ?$ Set "s" on the B scale over "r" on the D scale. Move the hairline to the C index, and read the answer on the D scale.

For example:

$4.85/\sqrt{3.11} = ?$ Set the hairline over 4.85 on the D scale. Rotate the disk until 3.11 on the B scale is under the hairline. Move the hairline to the C index and read the answer 2.75 on the D scale.

$4.85/\sqrt{3.11} = 2.75$

Model Problem,

$r^2/s = ?$ Set "s" on the B scale over "r" on the D scale. Move the hairline to the C index and read the answer on the A scale.

For example:

$(3.56)^2/4.28 = ?$ Set the hairline to 3.56 on the D scale. Rotate the disk until 4.28 on the B scale is under the hairline. Move the hairline to the C index, and read the answer, 2.96, on the A scale.

$(3.56)^2/4.28 = 2.96$

THE DI SCALE

The DI (pronounced Dee-Eye) scale found on Models 109 ES and 111 ES is the inverse of the D scale, just as CI is the inverse of the C scale. As such, the DI scale may be used to make particular calculations with a minimum of settings as shown below.

Model problem: $r/\sqrt{s}$

Set "s" on the A scale under "r" on the C scale. Move the hairline to the index line of the C scale, flip the scale and read the answer under the hairline on the DI scale.

For example:

$4.85/\sqrt{3.11} = ?$

Set the hairline over 3.11 on the A scale. Bring 4.85 on the C scale under the same hairline. Move the hairline to the C index, and read 2.75 under the hairline on the DI scale on the rear face of the slide rule.

$4.85/\sqrt{3.11} = 2.75$

Model problem: $\frac{r^2 s}{\sqrt{t}}$

Set "s" on the C scale over "t" on the A scale. Move the hairline to "r" on the EI scale, and read the answer on the DI scale.

For example:

$\frac{(2.37)^2 \times 4.62}{\sqrt{75.6}} = ?$

Set the hairline on 75.6 on the A scale. Bring 4.62 on the C scale under the hairline. Move the hairline to 2.37 on the EI scale, flip the slide rule to the back face and read the answer on the DI scale as 2.98.

$\frac{(2.37)^2 \times 4.62}{\sqrt{75.6}} = 2.98$

Many other combinations are possible. As you gain skill in recognizing and using reciprocal relationships the inverse scales will become increasingly useful to you.

THE 50-INCH EXPANDED C AND D SCALES

On the back face of the Model 110 ES are a pair of spiral, expanded, C and D scales which encircle the slide rule four times, giving a set of scales measuring more than 50 inches long. The increased length of these expanded C and D scales allows for much more precise setting of values, leading to calculations which in readings may be made to five significant figures in some parts of the scale at the "low" end, and never less than three significant figures in the "high" end of the scale. Throughout most of the scale, readings of four significant figures are easily and precisely made. These are the scales to use for work involving the need of an additional place beyond the usual 3 significant figures of the usual 10-inch slide rule.

Notice that there is an actual division mark for the first three significant numbers throughout the scales, and the divisions are adequately numbered, making it easy to set the desired numbers. The index line of these two expanded scales is indicated by the two triangles with facing apexes, and is best set by using the hairline, since the index line is not continuous.

Also, notice that at the index line, the scales are "spiraled" to give them their more than fifty-inch lengths. These expanded C and D scales are best read at the bottom or 6 o'clock position of the slide rule.

The expanded C and D scales may be used just as the regular C and D scales are used. Since it is important to read answers on the correct "cycle" of the expanded scale, it is best to first solve a problem on the regular C and D scales to determine the approximate answer, and then make the very precise settings that are possible on the expanded C and D scales. When it comes time to read your answer, you'll be able to select the correct section of the spiral scale because you already know the first couple of numbers in the answer.

For example:

$1.274 \times 3.678 = ?$

A rapid setting on the regular C and D scales shows us that 1.27 x 3.68 = 4.68, so we know the approximate answer to 3 significant figures. Now we go to the expanded D scale and locate 1.274 under the hairline. This number is located on the first, or inner, ring of the expanded D scale between the numbers 125 and 13 which are stamped on the scale. Now rotate the inner disk so that the index line is aligned with the hairline. Leaving the index set in this position, move the hairline to the setting of 3.678 on the expanded C scale. This number is on the third cycle of the expanded C scale. Now, under the hairline, on the expanded D scale read the precise answer on the third cycle of the expanded D scale as 4.689. We knew that our answer was close to 4.68, so we looked for the cycle that had this value close to the hairline (it was the third one) and then read the precise setting of 4.689 as the answer to four significant figures.

$1.274 \times 3.678 = 4.689$

For example: $\frac{8.467}{37.94} = ?$

A quick solution on the regular C and D scales shows us that 8.47/37.9 = .223 to 3 significant figures. Now, going to the expanded D scale on the back face of the slide rule, we set 8.467 under the hairline (it's on the fourth or outer cycle). Rotating the inner disk, bring 37.94 on the third cycle of the expanded C scale under the same hairline. Now move the hairline to the index line of the expanded C scale and locate the precise answer on the expanded D scale. We know it is .223, so we read the second cycle setting and find it to be .2232 to four significant figures.

In a similar manner, any operation of the regular C and D scales, such as multiplication, division, proportions, and so forth, may be performed on the expanded C and D scales when a greater precision or an additional significant figure is needed.

STADIA CALCULATION FROM SURVEYING USING THE G SCALE OF MODEL 110 ES

In stadia surveying, the horizontal distance and the level difference between two survey stations are expressed by the following formulas:

For horizontal distance: $D = (k\ 1\ Cos^2) + (c\ Cos\ A)$
For level difference: $H = (k\ 1\ Sin\ A\ Cos\ A) + (c\ SinA)$
where
k = stadia multiplier (which usually has a value of 100)
1 = distance between upper and lower visible points of staff
c = stadia number (a correction factor) which varies from instrument to instrument
A = angle of inclination (elevation or depression)

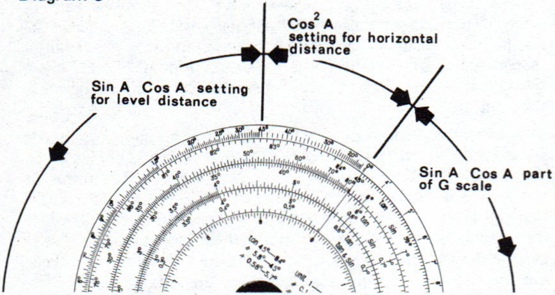
The value of "1" and angle "A" being found by actual measurement, the values for H and D may then be calculated.

The second part of each of these formulas, the (c Cos A), or the (c Sin A), can be easily calculated using the S, C, and D scales of the slide rule, as already covered. The more complicated first parts of these two formulas, that is, (k 1 Cos^2 A) and (k 1 Sin A Cos A) may also be calculated using the trigonometric scales of the slide rule, but your Model 110 ES has a special scale that greatly facilitates this calculation, namely, the G scale.

This G scale which is on the back face of the slide rule needs some study in order to properly set the observed angles on it. The G scale is best read at the 12 o'clock position. The index of the S scale coincides with the indexes of the other scales and is marked as 0^{o}. As you follow the G scale around in a clockwise direction, notice that its readings increase, and that it is graduated in degrees and minutes, not in decimal parts of a degree as are the sine and tangent scales. Follow the readings clockwise, around the scale until you come to 45^{o}. You will be within an inch or so of completing a complete circuit of the G scale. As you continue on around the G scale past 45^{o}, note that the scale becomes inverse, and the values start to decrease, going back to 40^{o}, then 30^{o}, 20^{o}, 10^{o} (which is not printed) and finally back to 0^{o} at the G scale index line.

The 45^{o} (high-water) mark is important. It marks the dividing point for setting the A angle. In solving for the horizontal distance, using (K 1 Cos^2A), the angle A is set to the right of the 45^{o} mark. In solving for the level distance, using (K1 Sin A Cos A), the angle A is set to the left of the 45^{o} mark. The diagram on Page 62 may help clarify this special setting.

Diagram 8



The following example will illustrate a typical stadia computation.

Suppose that: $k = 100$
$c = .30$ meters
$1 = 1.6$ meters
Angle $A = 18.5^{o}$

We are to calculate the horizontal distance, D, and the level difference, H. First, to calculate the horizontal distance, D:

$D = Kl\ (Cos^2 A) + c(Cos A)$

$D = 160(Cos^2\ 18.5^{o}) + .3(Cos\ 18.5^{o})$

Calculate k1 as $100 \times 1.6 = 160$. Set the hairline over 160 on the K scale on the front face of the slide rule. Turn the slide rule over to the back side, and rotate the inner disk until the index line of the G scale is under the hairline. Now move the hairline to the "high-water" mark of 45° on the G scale. Since we are to calculate the Cos^2 A part of the distance formula, we will set our angle A in the space to the right of the 45° mark on the G scale, at 18.5°. Note that this part of the scale the readings decrease. Now, flip back to the front side of the slide rule, and read the answer to (160) $\mathrm{Cos}^2\ 18.5^{\circ}$ under the hairline on the K scale as 143.9.

$D\ H = 143.9 + .3\ \mathrm{Cos}\ 18.5^{\circ}$

As covered previously, determine Cos 18.5° from the S and D scales, multiply it by .3, giving you .28.

$D\ H = 143.9 + .28 =$ 144.2 meters

Now, to calculate the level difference, H:

$H = k1\ \mathrm{SinA}\ \mathrm{CosA} + c\ \mathrm{SinA}$

We'll calculate the (k1SinACosA) term first. Calculate k1 as 100 x 1.6 = 160.

$H = 160\ \mathrm{Sin}\ 18.5^{\circ}\ \mathrm{Cos}\ 18.5^{\circ} + \mathrm{Sin}\ 18.5^{\circ}$

Set the hairline over 160 of the K scale on the front side of the slide rule. Turn the slide rule to the back side and rotate the inner disk until the index line of the G scale is under the hairline. Now move the hairline to the "high-water" mark of 45° on the G scale. Since we are to calculate the SinACosA part of the level difference formula, we will set our angle A in the space to the left of the 45° mark on the G scale, at

$H = 48.1 + .3$
Sine 18.5^o

$H = 48.1\ M + .10$
$H = 48.1 + .10 =$
48.2 meters

18.5^o. Move the hairline to the left of 45^o to the 18.5^o value on the G scale. Flip the slide rule over to read the answer to (160Sin 18.5^oCos 18.5^o) on the K scale as 48.1. Now, getting Sin 18.5^o and multiplying by .3, as covered previously, we get .10. Add these two parts together to get the final answer, 48.2 meters.

To summarize this problem, then, D = 144.2 meters, H = 48.2 meters. The only tricky part of this stadia problem is the use of the G scale for the Cos^2A and SinACosA calculations. To give you practice in M using the G scale, then, the following problems and answers are given:

50 meters x $Cos^2 15^o$ = 46.7 meters
230 meters x $Cos^2 28^o$ = 179.3 meters
50 meters x Sin 15^o Cos 15^o = 12.5 meters
230 meters x Sin 15^o Cos 15^o = 95.3 meters